Jake Traut

1/26/16

CSCI 3656 Numerical Computation Problem Set 2

1. Bisection method for equation f(x) = sin(x) + 0.25 with initial guesses x = -1 and x = 1.
   * Number of iterations to get f(x) < 0.0001 = 12 (final iteration at 5.1237e-06). Error decreases with linear convergence, just about halving at each step (with exception of first 3 iteration).
   * Initial guesses changed to x = -1 and x = 0. Now number of iterations = 11, and the error goes up after the first iteration then gradually back down. This means our initial guess here was closer to bracketing the root than the first bisection (iteration), then followed linearly after.
   * New guesses.
     1. Guesses x = 3 and x = 4: root = 3.3943
     2. Guesses x = 4 and x = 7: root = 6.0305
2. Secant method with initial guesses x = -1 and x = 1: number of iterations = 2 (with more error than bisection)
3. Newton’s method with initial guess x = -1: number of iterations = 2 (with smaller error than secant and bisection)
4. The convergence patterns of these three methods are consistent with what we know. The bisection method took the most work with 12 iterations and a linear convergence, essentially halved at each step, while secant only took 2 iterations to reach an error < 0.0001, a superlinear convergence rate with 1 < exp < 2, but this also resulted in a larger error than bisection for these specifications, jumping from 0.0043 to 2.6224e-05 (compared to 5.1237e-06). Lastly, newton’s method follows the expected quadratic convergence rate at exp = 2 with error jumping from 0.0012 to 2.0114e-07 on 2nd iteration, resulting in least steps with highest accuracy.
5. Fixed point method: initial guess x = 1 gives that the root is 1, but… Using initial guess 1.2 and tolerance = 0.0001
   * Rearrangement 1: x = x^3 – 3x^2 + 3 is divergent, an unstable function that despite any number of iterations it doesn’t move towards a specific limit, makes no progress.
   * Rearrangement 2: x = (3x^2 + x – 3)^(1/3) is convergent but at a very slow rate, crawling its way from 1.5763 to 3 over the course of 36 steps.
   * Rearrangement 3: x = ((x^3 – x + 3)/3)^(1/2) converges to 1, also by small increments per step but being closer to 1 to start it takes 8 iterations.
   * Thus the two convergent functions give us the roots 1 and 3.